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Please do not open this question sheet until you are told to do so.

4810-1183 Approximation and Online Algorithms with Applications
Spring 2018 – Final Examination

Problem 1

In the problem 1 of our midterm examination, we have worked on the following optimization model:

Input: $\mathcal{A}$

Output: $\mathcal{B}$

Constraint: $\mathcal{C}$

Objective Function: Maximize $\mathcal{D}$

We assume that, for any output that satisfies the constraint $\mathcal{C}$, the objective value $\mathcal{D}$ is non-negative. Let us call the above optimization model as **Problem 1**.

Question 1.1: Define an optimization model that is as hard as having a 0.5-approximation algorithm for Problem 1.

From now, we will assume that it is not possible to have a 0.5-approximation algorithm for Problem 1. Let us recall an optimization model with the same input, output, and constraint, but with the following objective function.

Objective Function: Minimize $1/\mathcal{D}$

We will call the optimization model as **Problem 2**.

Question 1.2: Define an optimization model that is as hard as having a 2-approximation algorithm for Problem 2.

Question 1.3: Assume that there is a problem for your optimization model in Question 1.2. Write a program to solve an optimization model in Question 1.1. Discuss why your program works.

Question 1.4: Based on your answer in Question 1.3, discuss why it is not possible to have a 2-approximation algorithm for Problem 2.

From the following question, let us assume that, for any output that satisfies the constraint $\mathcal{C}$, the objective value $\mathcal{D}$ is between 0 and 1. Consider **Problem 3** with the same input, output, and objective function, but with the following objective function.

Objective Function: Minimize $1 - D$

Question 1.5: Assume that we have a 2-approximation algorithm for problem 3. If we use that algorithm for Problem 1, what are the guarantees on the outputs' objective values when the optimal value (of Problem 1) are 0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, and 2.

Question 1.6: Based on your answer in Question 1.5, discuss why knowing that "an α-approximation algorithm is not possible for problem 1" does not help us showing that "an α'-approximation algorithm is not possible for problem 3".

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Problem 2

In this problem, we will introduce the concept of randomized algorithm to the secretary problem. Recall that the formulation of the problem is as follows:

Input: Score of each secretary $s_1, \ldots, s_n \in \mathbb{R}_{\geq 0}$

Output: $d_1, \ldots, d_n \in \{\text{Yes}, \text{No}\}$: d_i is yes if i is selected, otherwise d_i is no.

Constraint: 1) Only one d_i is yes
2) d_i is decided based on only $s_1, \ldots s_i$

Objective Function: Maximize the score of the selected participant

We will select the secretary i with probability i/n, if s_i is the largest among $s_1, \ldots, s_i$.

<u>Question 2.1</u>: Discuss why the probability that we choose applicant 1 is $1/n$. Also, discuss why the probability that we <u>correctly</u> choose applicant 1 is $1/n^2$.

<u>Question 2.2</u>: Discuss why the probability that we choose applicant 2 is $\frac{n-1}{n^2}$. Also, discuss why the probability that we correctly choose applicant 2 is $\frac{2}{n} \cdot (n-1)/n^2$.

<u>Question 2.3</u>: Discuss why the probability that we choose applicant 3 is $(n-1)^2/n^3$. Also, discuss why the probability that we correctly choose applicant 3 is $\frac{3}{n} \cdot (n-1)^2/n^3$.

<u>Question 2.4</u>: From your answer of Question 2.1 – 2.3, guess what the success probability of this strategy is.

<u>Question 2.5</u>: Given $\sum_{m=1}^{n} \left[\left(\frac{n-1}{n} \right)^m \cdot m \right] = (n-1)n^{1-n}[n^n - 2(n-1)^n]$. Write your success probability Question 2.4 in a form without summation.

<u>Question 2.6</u>: Given $\lim_{n\to\infty} 1 - 2\left[\frac{n-1}{n}\right]^n = 1 - \frac{2}{e}$. Discuss why this strategy is not better than the strategy discussed in the class.

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Problem 3

Online learning platforms, like Coursera or EdX, were expected to replace conventional educations a few years ago. However, they are failed to do so. The dropout rate of each course is very large, and students are not very motivated to continue.

An insufficient communication between students could cause that large dropout rates. There are several communication methods such as forums or Slack, but they still cannot replace in-person communications.

The platforms now recommend the courses to each user, but we will change their advertising strategy in this problem. We will assume that they recommend courses to a friendship. Instead of telling Alice courses that she might interested, we will tell Alice and Bob, who are friends of each other, the courses that they might interested in learning together.

We want to maximize the number of times we recommend a particular course to friends. For simplicity, you can assume that we want to maximize the sum of the frequency. However, we do not want to recommend too many problems to a particular person, otherwise he/she may consider our recommendations as spams.

From the following questions, let us formulize the above situation to a mathematical model.

Question 3.1: What is the input of your optimization model?

Question 3.2: What is the output of your optimization model?

Question 3.3: What is the objective function of your optimization model?

Question 3.4: What is the constraint of your optimization model?

Question 3.5: Assume that the number of courses can be real numbers. Discuss how you can use a library for linear programs to solve your optimization model.

Question 3.6: Discuss how you can use the randomized rounding technique to solve your optimization model.

Question 3.7: Using Markov's inequality, what is the probability that your solution from Question 3.6 satisfies the constraint in Question 3.3.

Question 3.8: What is the approximation ratio of your randomized rounding technique in Question 3.6?